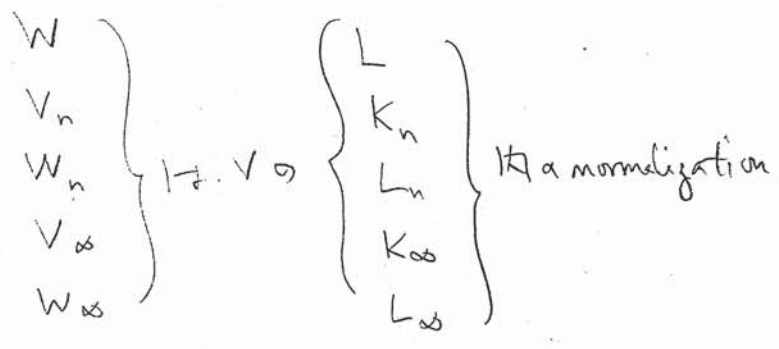


Thm: V DVR, mixed char., residue fld. = k , quotient fld. = K .
 $t_1, \dots, t_r \in V$ s.t. $t_i \mapsto \bar{t}_i \in k$ とする.

$L/K < \infty$
 $K_n := K(t_1^{1/p^n}, \dots, t_r^{1/p^n})$, ($c = \text{constant} \in \mathbb{N}_{\geq 1}$ は後で選ぶ)
 $L_n := L \cdot K_n$
 $K_\infty := \bigcup_n K_n$, $L_\infty := \bigcup_n L_n$

$k/k_0(\bar{t}_1, \dots, \bar{t}_r)$ fin., separable;
 k_0 perfect
 ↑ ... ↑
 不定元



\Rightarrow W_∞/V_∞ H almost étale.

Proof: ① $V' := V[t_1^{1/p^\infty}, \dots, t_r^{1/p^\infty}]$: DVR, mixed char., res. fld. perfect

↓

$V \sim V'^{1=1, 2} \quad \boxed{k = k_0, r = 0}$ とし

$\sim (V')^{\wedge 1=1, 2} \quad \boxed{V \text{ complete}}$ とし

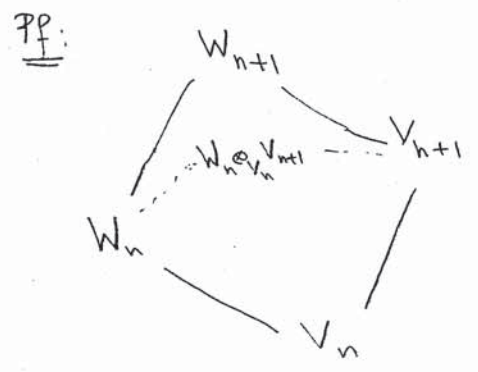
(以下2つは ξ_0 と仮定する)

② Lemma: $\Omega_{W/V} \cong W/p^\delta W$ ($p^\delta: \text{ord}(-) = \delta \cdot \text{ord}(p)$ と (1) と (2) と $\bar{\pi}$, $\delta \in \mathbb{Z}_{\geq 0}$)
 且 $p^\delta W = \text{'different'}$ (cf. ① と (2) と (3))

PF: $\pi_W: W$ a uniformizer と (3) と. $W \cong V[\pi_W] \cong V[X]/(f(X))$
 $\Rightarrow \Omega_{W/V} \cong (W \cdot dX) / (f'(X)dX) \cong W/p^\delta W$
 ... $p^\delta W = \text{'different'}$, 且 $\delta < \text{ord}(f'(X))$ (cf. Serre, Local Fields)

(c と (1) と (2) と (3) と)

③ Claim: $\forall n$, 十分大. $\Omega_{W_{n+1}/V_{n+1}} \cong \Omega_{W_{n+1}/(W_n \otimes_{V_n} V_{n+1})}$



一般論より.

$$\begin{aligned} \Omega_{(W_n \otimes_{V_n} V_{n+1})/V_{n+1}} &\xrightarrow{\varphi} \Omega_{W_{n+1}/V_{n+1}} \\ (\Omega_{W_n/V_n} \otimes_{V_n} V_{n+1}) &\xrightarrow{\psi} \Omega_{W_{n+1}/(W_n \otimes_{V_n} V_{n+1})} \rightarrow 0 \end{aligned}$$

一方, (2) と $W_{n+1}, V_{n+1}, W_n, V_n$ 等々適用 (3) と

は完全.

$$\left. \begin{array}{l} \Omega_{W_{n+1}/V_n} \\ \Omega_{V_{n+1}/V_n} \otimes_{V_n} W_{n+1} \\ W_{n+1}/p^\delta W_{n+1} \end{array} \right\} \cong \left. \begin{array}{l} \Omega_{W_n/V_n} \otimes_{W_n} W_{n+1} \\ W_{n+1}/p^\delta W_{n+1} \end{array} \right\}$$

$W_{n+1}/p^\delta W_{n+1}$ と書く.

(ii) $\delta \geq \dots \geq \delta_{n+1} \geq \delta_n$ (簡単な演習問題)
 従って $c \geq \delta$ と (1) と (2) と (3) と $c \geq \text{ord}(f'(X))$ (cf. ④) $\Rightarrow \varphi = 0 \Rightarrow \psi$ は \cong

④ Claim: $(\phi^{\delta_n - \delta_{n+1}}) \cdot W_{n+1} \cong W_n \otimes_{V_n} V_{n+1} (\cong W_{n+1})$

Pf: $\underbrace{\phi^{-\delta_{n+1}} \cdot W_{n+1}}_{\cong M} \cong \underbrace{\phi^{-\delta_n} \cdot (W_n \otimes_{V_n} V_{n+1})}_{\cong N}$ 示せばOK.

- $\bar{\omega}$, (different a Tr. 系を定義せよ) $\text{Tr} := \text{Tr}_{L_{n+1}/K_{n+1}}$ によれば,

$$\text{Tr}((W_n \otimes_{V_n} V_{n+1}) \cdot M) \subseteq \text{Tr}(W_{n+1} \cdot M) \subseteq V_{n+1}$$

(V_{n+1}/V_n flat 故)
 $\Rightarrow M \subseteq N //$

⑤ Lemma: $A \subseteq B \subseteq C$ commutative rings, $\partial 1$; $B, C : A$ -flat
 $\gamma \in C$ st. $\gamma \cdot C \subseteq B$

\Downarrow
 $\boxed{\gamma^2 \cdot \Omega_{C/B} = 0}$

Pf:
$$\begin{array}{ccccccc} 0 \rightarrow & I_B & \rightarrow & B \otimes_A B & \rightarrow & B & \rightarrow 0 \\ & \downarrow & & \downarrow & & \downarrow & \\ 0 \rightarrow & I_C & \rightarrow & C \otimes_A C & \rightarrow & C & \rightarrow 0 \end{array} \quad \left. \vphantom{\begin{array}{ccccccc} 0 \rightarrow & I_B & \rightarrow & B \otimes_A B & \rightarrow & B & \rightarrow 0 \\ & \downarrow & & \downarrow & & \downarrow & \\ 0 \rightarrow & I_C & \rightarrow & C \otimes_A C & \rightarrow & C & \rightarrow 0 \end{array}} \right\} \text{exact}$$

$\Rightarrow (\gamma \otimes \gamma)(C \otimes_A C) \subseteq B \otimes_A B, I_C \cap (B \otimes_A B) = I_B \nexists 1)$

$\boxed{(\gamma \otimes \gamma) \cdot I_C \subseteq I_B}$

$\Rightarrow C \otimes_A (I_B/I_B^2) \rightarrow I_C/I_C^2 \rightarrow \Omega_{C/B}$ (exact: 一般論)

$C \otimes_A (\Omega_{B/A}) \quad \Omega_{C/A} \quad \nexists 1), \Omega_{C/B} \cong I_C / (I_C^2 + C \cdot I_B)$

$\Rightarrow (\gamma \otimes \gamma) \cdot (I_C/I_C^2 + C \cdot I_B) = \gamma^2 \cdot (I_C/I_C^2 + C \cdot I_B) = 0 //$

(p 4)

$$\textcircled{6} \quad \textcircled{4} \text{ \& } \textcircled{5} \text{ f } \mathbb{F} \lambda \Rightarrow p^{2(\delta_n - \delta_{n+1})} \cdot \sum W_{n+1} / (W_n \otimes V_{n+1}) = 0$$

$$\Rightarrow (\textcircled{3} \text{ f } 1)$$

$$p^{2(\delta_n - \delta_{n+1})} \cdot \underbrace{\sum W_{n+1} / V_{n+1}}_{\substack{||2 \\ W_{n+1} / p^{\delta_{n+1}} W_{n+1}}} = 0.$$

$$\Rightarrow \delta_{n+1} \leq 2(\delta_n - \delta_{n+1})$$

$$\Rightarrow 3\delta_{n+1} \leq 2\delta_n$$

$$\Rightarrow \delta_{n+1} \leq \frac{2}{3} \delta_n$$

$$\Rightarrow \boxed{\lim_{n \rightarrow \infty} \delta_n = 0.}$$

... \Rightarrow f 1), W_∞ / V_∞ is almost étale